

Mechanical models of cellular solids: parameters identification from experimental tests - part II

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Mechanical models of cellular solids: parameters identification from experimental tests - part II / Avalue, Massimiliano; Belingardi, Giovanni; Ibba, A.. - 1:(2005). (Intervento presentato al convegno 19th International Technical Conference on the Enhanced Safety of Vehicles - ESV).

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# MECHANICAL MODELS OF CELLULAR SOLIDS, PARAMETERS IDENTIFICATION FROM EXPERIMENTAL TESTS

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Paper Number 05-0438

## ABSTRACT

Cellular solids are largely used in many structural applications to absorb and dissipate energy, due to their light weight and high energy absorption capability.

The appropriate design of mechanical pieces made of structural foams must be done on the basis of the kind of impact, the energy involved and the maximum admissible stress. In the design development it is of highest importance the choice of the proper type of foam at the proper density level. This is based on stress-strain behaviour that can be predicted by means of test curves and models.

The parameters of two cellular solids models for EPP, PUR, EPS and NORYL GTX foams have been identified by means of experimental compression tests at different densities. The Gibson model and a modified version of this model have been considered: the fitting of these models are compared also with the Rusch model and a modified version of the Rusch model.

The considered models are directly derived from theoretical micro-mechanical assumptions while the parameter values are identified by means of the available experimental data.

Model parameters depend on the foam density and a mathematical formulation of this dependence is identified.

The formulas for the density dependence of the model parameters permits the identification of all foams made starting from the same solid material and with the same micro-structure by means of a minimum set of experimental tests. At the same time the availability of a large quantity of experimental data allows to reach a higher confidence level for the model parameters values.

The identified laws that describe parameters against density, for a certain type of foam, could be used in order to assist the design of the absorber and to find the optimum density for the specific application.

## 1. INTRODUCTION

The increasing request of more performing and safer vehicles has given great importance to cellular solid materials in automotive industry. This kind of materials is successfully used in vehicles in order to minimise the weight of structural components and to improve the safety through the absorption of impact energy in crash events.

In this second type of application cellular solids are used to absorb impacts between vehicle and external obstacles and the consequent internal impacts of the passengers against the body structure of the vehicle. In both cases cellular solid components should be designed in order to control the deceleration of the impacting parts (vehicle or passengers) and in order to limit its maximum value. For this aim cellular solids are very functional and permit the design of a component to meet the exact requirements for a specific impact. The mechanical characteristics can be modulated in a continuous way acting on the density and on the micro-structure besides on the constitutive material: it is possible to integrate the design of the proper mechanical characteristics with the design of shape and dimensions of the component.

This functionality implies more complexity in design computation. It could be very onerous to characterise and identify all cellular solids which can be used for a specific application and it could be onerous to analyse the behaviour of each of them.

For design purposes a unified modelisation of a larger set of materials identified with less experimental testing could be very helpful. A unique model for cellular solids made of the same constitutive material and for a wide range of density could be used to direct the choice of the optimal foam density for defined impact energy absorption. Moreover it can be used in FEM crash simulations in order to analyse the behaviour of different foams, also experimentally not tested. The aim of this work is the analysis and development of these features on cellular solids modelisation.

Experimental testing on four types of foams have been analysed and different models have been identified and compared by means of these experimental data.

The variation of model parameters with foam density has been studied in order to develop parameters-density laws to be integrated in each model.

## 2. EXPERIMENTAL TESTS

Static uniaxial compression tests, made according with ASTM D1621-94 (*Standard Test Method for Compressive Properties of Rigid Cellular Plastics*) have been performed on different kinds of foams at different densities:

- **Expanded polypropylene** foams (EPP) tested at five different nominal densities with a wide range of variation: 31, 45, 70, 106 and 145 g/dm<sup>3</sup>.
- **Expanded polystyrene** foams (EPS) tested at four different nominal densities, but the range of variation is relatively narrow: 40, 50, 60 and 70 g/dm<sup>3</sup>.
- **Expanded polyurethane** foams (PUR) tested only at two different densities: 70 and 100 g/dm<sup>3</sup>.
- **Noryl GTX** foams, tested at two different densities: 50 and 75 g/dm<sup>3</sup>.

All specimens were cubic with 50 mm side, except for the expanded polystyrene specimens which were cylindrical with a diameter of 100 mm and height of 35 mm. Each specimen was previously weighted and measured in order to calculate its effective density. The tests consist of the compression of the foam specimen between two rigid steel plates at a constant relative velocity of 60 mm/min, which corresponds to a strain rate of  $2 \times 10^{-2} \text{ s}^{-1}$ . The maximum stroke chosen is 90% of the initial thickness. A hydraulic universal testing machine (DARTEC 9600) was used; the piston displacement and the force were measured at an appropriate sampling frequency. For each nominal density of each kind of foam at least three repetitions of the compression test were performed.

## 3. CELLULAR SOLIDS MODELS

Cellular solids models can be divided in two categories: phenomenological model and micro-mechanical models. The phenomenological models aim to reach the best fit with the experimental mechanical behaviour without direct relationship with the physics of the phenomenon. The micro-mechanical models are based on the analysis of the

deformation mechanisms of the micro-cell structure under loading.

The micromechanical Gibson model and a proposed modified Gibson model have been identified for all tested materials. Their fitting capability has been compared with the phenomenological Rusch model and a modified Rusch model already identified for the same foams in [6].

### 3.1 Gibson Model

The most known and widely used micro-mechanical model is the Gibson model (Gibson, Ashby [1]) in which the stress-strain compression curve is split into three parts (elastic, collapse and densification) and analytical relationships are obtained. The behaviour is mainly controlled by the relative density of the foam with respect to the solid base material.

The formulations of the three regions are:

- Linear elastic region:

$$\mathbf{s} = E\mathbf{e} \quad (1a)$$

$$\text{if } \mathbf{s} \leq \mathbf{s}_{yield} \quad (1b)$$

- Plateau region:

$$\mathbf{s} = \mathbf{s}_{yield} \quad (2a)$$

$$\text{if } \mathbf{e}_{yield} \leq \mathbf{e} \leq \mathbf{e}_D (1 - D^{-1/m}) \quad (2b)$$

- Densification region:

$$\mathbf{s} = \mathbf{s}_{yield} \frac{1}{D} \left( \frac{\mathbf{e}_D}{\mathbf{e}_D - \mathbf{e}} \right)^m \quad (3a)$$

$$\text{if } \mathbf{e} > \mathbf{e}_D (1 - D^{-1/m}) \quad (3b)$$

The model has five parameters and each of them can be calculated by means of author's dedicated formulas based on the micro-mechanics of foam deformation. In this work the parameters have been identified on the basis of the experimental curves so that the identified values can be compared to the theoretical ones. The parameters  $E$ ,  $\sigma_{yield}$  and  $\epsilon_D$  are considered density dependent, while  $D$  and  $m$  should be density independent.

### 3.2 Modified Gibson Model

The original Gibson model has been modified in order to test the possibility to improve its fitting capability. A sloped linear model is proposed instead

of a constant stress model for the modelling of the plateau region. The equation (2a) is substituted by the following expression:

$$\mathbf{s} = \mathbf{s}_{yield} + h\mathbf{e} \quad (4)$$

This modification implies the identification of two parameters instead of a single one for this region, and a total of 6 parameters for the whole model.

The strain value of the intersection between the plateau region and the densification region can not be expressed explicitly as in the original Gibson model: it must be found numerically.

### 3.3 Rusch Model and Modified Rusch Model

A simpler and effective phenomenological model is the Rusch model (Rusch [2], [3] and [4]). It had been extensively tested in previous works of the authors (Avalle, Belingardi, Ibba [6]) and it demonstrated a good fitting capability combined to some advantages in the identification process with respect to the Gibson model.

A modification of this model had also been proposed in order to improve the fitting capability in the densification region. Both models had been identified in [6] for the same foams analysed here.

The Rusch model is a phenomenological model having a simple formulation, described by the sum of two power laws:

$$\mathbf{s} = A\mathbf{e}^m + B\mathbf{e}^n \quad (5a)$$

$$\text{with } 0 < m < 1, 1 < n < \infty \quad (5b)$$

$\mathbf{e}$  is engineering strain and is considered positive in compression. The first addendum is used to fit the elastic-plateau region, while the second addendum is used to model the densification region. The parameters  $A$  and  $B$  are considered density dependent, while  $m$  and  $n$  are not.

The modified version of the Rusch model, proposed in [6], had been developed in order to improve the fit in the densification region. The second addendum is modified in order to have a vertical asymptote at the physical limit of compression strain:

$$\mathbf{s} = A\mathbf{e}^m + B\left(\frac{\mathbf{e}}{1-\mathbf{e}}\right)^n \quad (6a)$$

$$\text{with } 0 < m < 1 \text{ and } 1 < n < \infty \quad (6b)$$

## 4. IDENTIFICATION METHOD

Two procedures of parameter identification have been used in order to identify the material parameters, for each model:

1. The whole set of parameters of the models are identified for each experimental curve
2. Only the density dependent parameters are identified for each experimental curve, while the density independent parameters are identified on the whole set of curves together

The second identification procedure is more interesting because it allows to separate the density influence from the other material parameters and to evaluate their dependence from the density itself. The least squares method has been used to identify the material parameters. However, it gave poor results when applied to the not weighted sum of the square errors, especially in the case of second procedure of identification. In fact, it suffers the over-weighted influence of the high density foams and vice versa. This is due to the fact that the gaps between experimental stress and model predicted stress of the low density foams are always low compared to the gaps of the higher density foams. Considering that the second kind of identification is obtained through the minimization of the sum of the square errors of all foam density curves, it is clear that the variation of the total sum due to the variation of the parameters of the low density foams is very little compared to the variation of the total sum due to the same relative variation of the parameters of the high density foams.

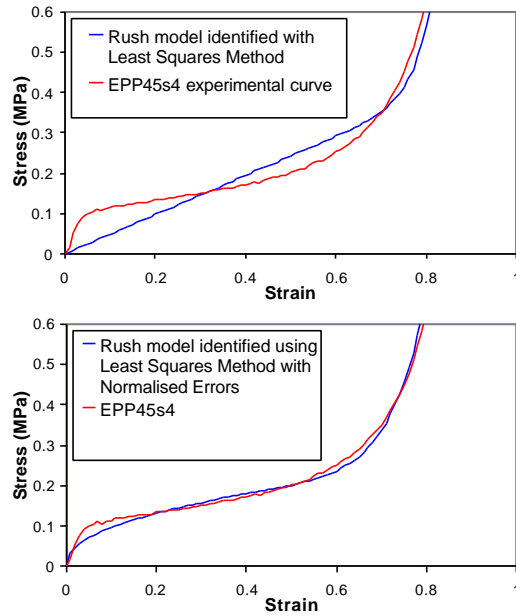
The identification based on plain sum of the square errors is too loose for low density foams at low strain (and stress). Therefore, the fitting has been performed by weighting the errors with the value of the experimentally measured stress. Hence, the sum of the normalised square errors (SNSE) to be minimised is:

$$SNSE = \sum_i \left( \frac{\mathbf{s}_{sper,i} - \mathbf{s}_{mod,i}}{\mathbf{s}_{sper,i}} \right)^2 \quad (7)$$

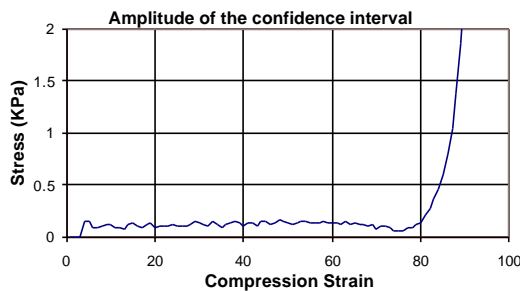
where  $\mathbf{s}_{sper,i}$  and  $\mathbf{s}_{mod,i}$  are the experimental stress and the model predicted stress respectively, corresponding to the same strain.

The normalised squares minimization procedure brings a better fit of the elastic and plateau regions although a slightly worse fit is obtained in the densification region (Figure 1).

The choice of the procedure based on the normalised least squares is justified by technical and statistical reasons.



**Figure 1. Comparison between the models identified with the least squares method using the squared errors and the normalised squared errors. Five densities of EPP foams have been identified at the same time.**



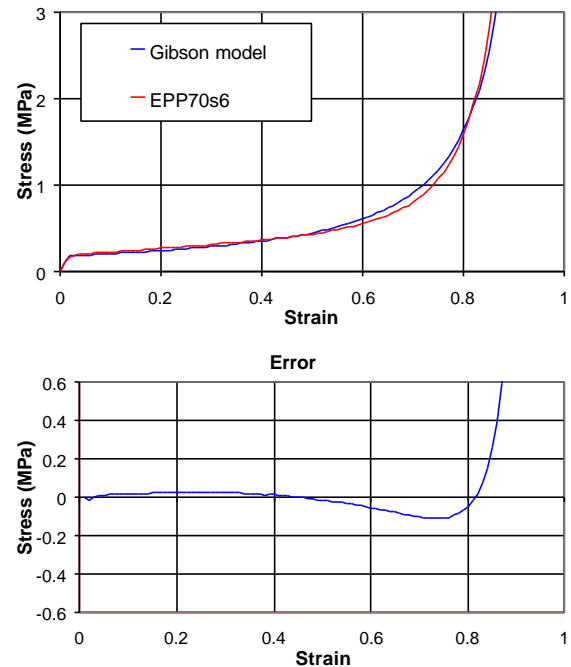
**Figure 2. Amplitude of the confidence interval versus strain.**

In impact applications, the foam should absorb a defined quantity of energy with fixed maximum displacement and stress level. This goal can be achieved by taking advantage of the linear and plateau region: from a design point of view the prediction of the energy involved in the plateau region has a primary role. The use of the foam in the densification region is unlikely because the foam would absorb energy with rising stress and quickly rising tangent stiffness. Moreover different densities of the same type of foam could result in large differences of the plateau stress, so in the global identification of all model curves it is advantageous to evaluate the gaps between the experimental curves and the model curves in a relative way: the errors should be proportional to the stress level of each curve.

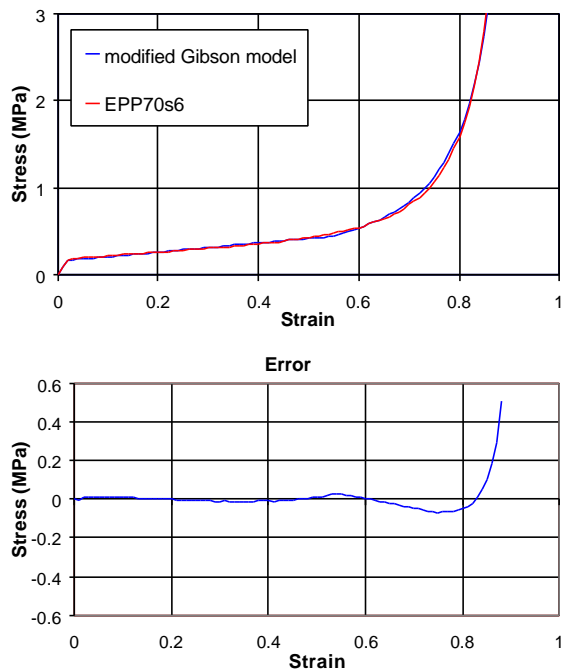
From a statistical point of view the lack of fit of the model must be evaluated weighting the effect of the variance of experimental data. The confidence intervals of the experimental stress values as function of the strain has been calculated by means of three repetitions of a test on the same foam. Figure 2 shows the remarkable quality of the fit in the elastic and plateau regions and the enlargement of the confidence intervals only in the densification region.

## 5. COMPARISON OF MODELS PERFORMANCE

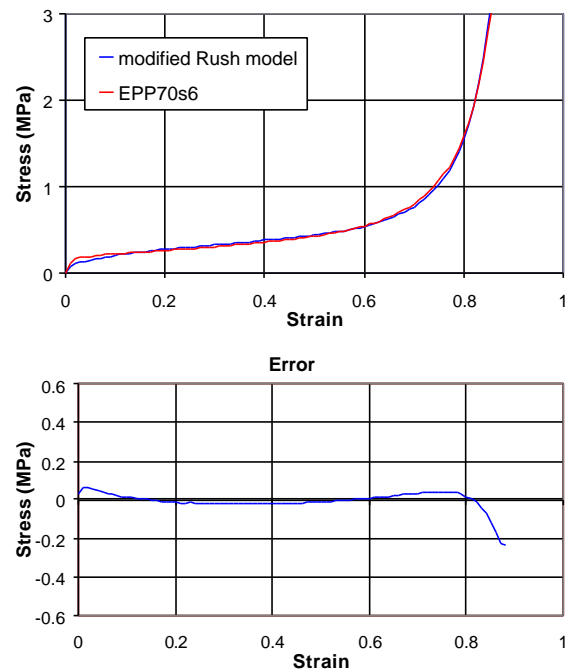
Performance of models can be visualised graphically by means of the stress-strain curves of identified models and experimental data. In Figures 3 and 4 the Gibson model and the modified Gibson model curves of the EPP at 70 g/l are shown, while in Figures 5 and 6 the Rusch model and the modified Rusch model identified in [6] for the same material are shown. All models seem to fit satisfactorily the experimental curves with an advantage for the Gibson type models. The difference between model stress and experimental stress (error) plotted as function of the strain is a better way to compare the fitting characteristics of models: these curves are shown for the same foam and the same models.



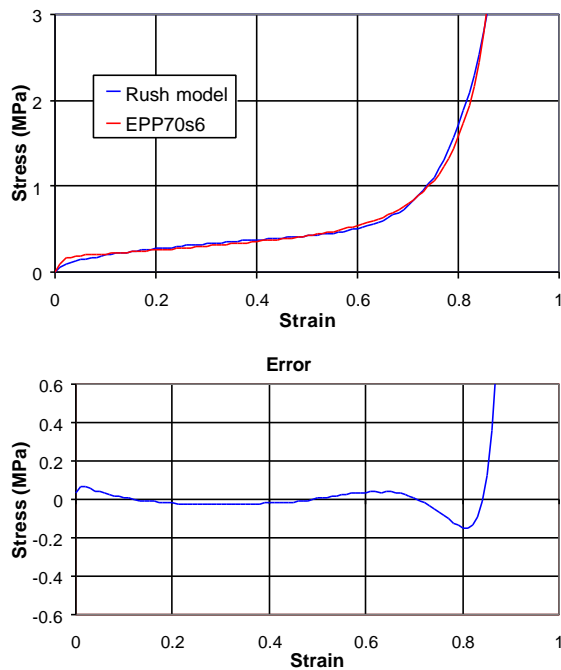
**Figure 3. Experimental stress-strain curve of the EPP 70 g/l and the corresponding identified Gibson model curve.**



**Figure 4.** Experimental stress-strain curve of the EPP 70 g/l compared to the modified Gibson model with the identified parameters.



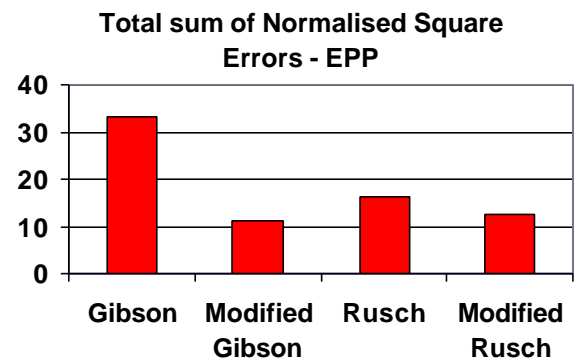
**Figure 6.** Experimental stress-strain curve of the EPP 70 g/l compared to the modified Rusch model with the identified parameters.



**Figure 5.** Experimental stress-strain curve of the EPP 70 g/l compared to the Rusch model with the identified parameters.

The identification has been performed with method 2 described in the previous paragraph. For this reason the fitting on each single curve is a little bit penalised by the need to have a unique identified value of the density independent parameters for the whole set of tested foams of the same type.

A quantitative comparison of the global fitting capability of the models has been performed by means of the total sum of the normalised square errors for each kind of foam (Fig. 7-10). The modified version of the Gibson model shows always the best fitting.



**Figure 7.** Comparison of the total sum of normalised square errors of the models identified for EPP foams.

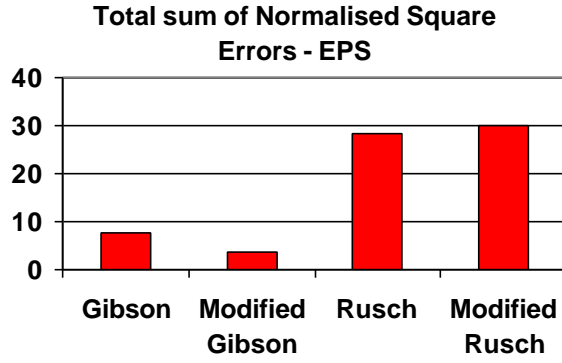


Figure 8. Comparison of the total sum of normalised square errors of the models identified for EPS foams.

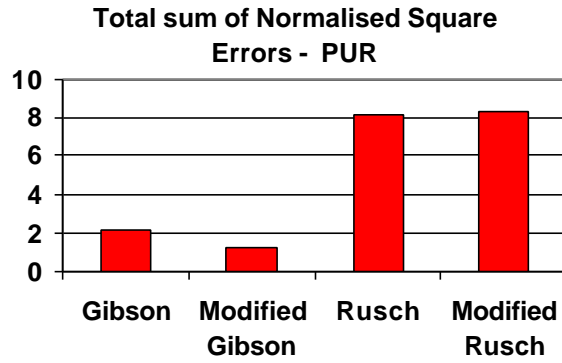


Figure 9. Comparison of the total sum of normalised square errors of the models identified for PUR foams.

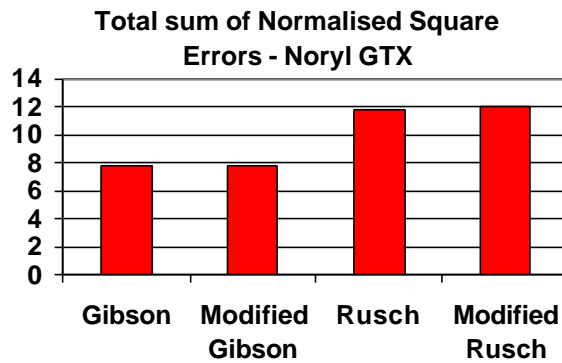


Figure 10. Comparison of the total sum of normalised square errors of the models identified for Noryl GTX foams

## 6. MODELS OF THE DENSITY EFFECT

The identified models have been further analysed in order to obtain the relationship between material

density and model parameters and to develop laws describing this source of variation. In the Gibson model, as a micromechanical model, this relationship is an assumption of the model itself, which can be eventually verified by comparison with the parameters values identified by experimental data.

### 6.1 Gibson Model

The Gibson model includes the density effect on parameters as a consequence of the micromechanical deformation mechanisms which are on the basis of the model itself. It does not need experimental data in order to quantify it: the micromechanical theory is used.

This feature of the Gibson model is useful when few experimental data on the density effect are available. On the other hand the availability of some experimental data on foamed materials at different density levels should be useful to better fit the effective density effect. For this aim the structure of the Gibson parameters-density laws have been maintained but its parameters have been identified through the experimental data.

#### Elastic Modulus

In case of open cell foams the variation of the elastic modulus with density is modelled by Gibson with the following relation:

$$\frac{E}{E_s} \approx \left( \frac{\mathbf{r}}{\mathbf{r}_s} \right)^2 \quad (8)$$

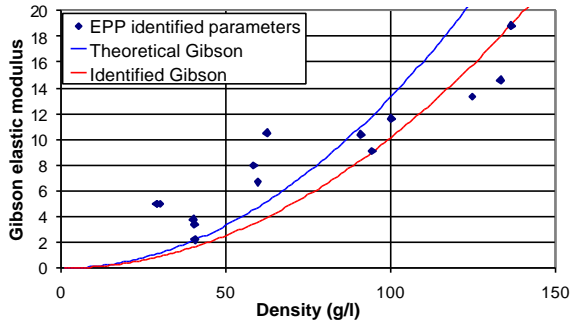
$E$ ,  $E_s$ ,  $\mathbf{r}$  and  $\mathbf{r}_s$  are the elastic modulus of the foam, the elastic modulus of the solid material of which the foam is made, the density of the foam and the density of the solid material respectively.

The equation can be written with a parameter to be identified by means of the elastic moduli already identified for each tested foam density:

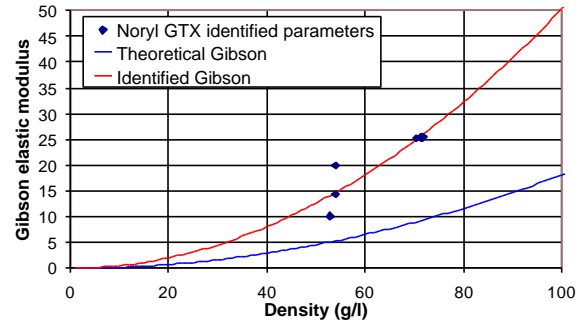
$$E = C_E \mathbf{r}^2 \quad (9)$$

In this relation the parameter  $C$  contains the ratio of the elastic modulus to the density of the solid material corrected by a factor which permits to reach a better fit of the experimental data.

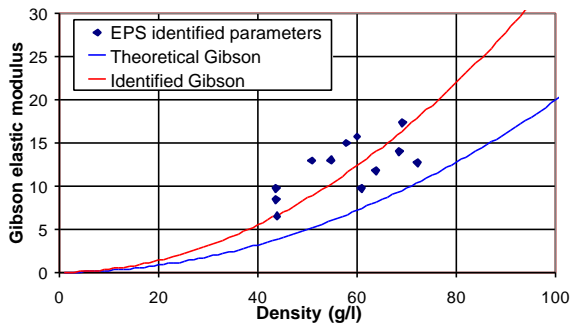
The experimentally identified elastic moduli for each specimen, the proposed relation, identified by means of these data, and the original Gibson relation are shown and compared in Figures 11-14 for each type of foam.



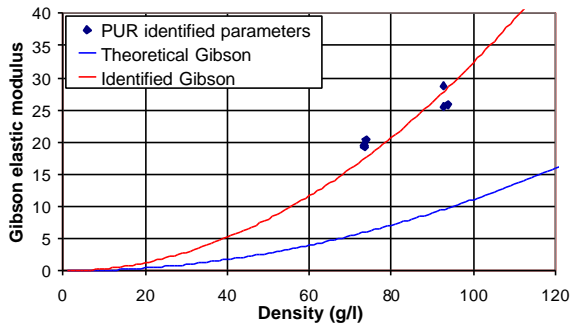
**Figure 11. Gibson model for the EPP foams: identified elastic moduli, theoretical parameter-density relation and identified relation.**



**Figure 14. Gibson model for the Noryl GTX foams: identified elastic moduli, theoretical parameter-density relation and identified relation.**



**Figure 12. Gibson model for the EPS foams: identified elastic moduli, theoretical parameter-density relation and identified relation.**



**Figure 13. Gibson model for the PUR foams: identified elastic moduli, theoretical parameter-density relation and identified relation.**

### Yield Stress

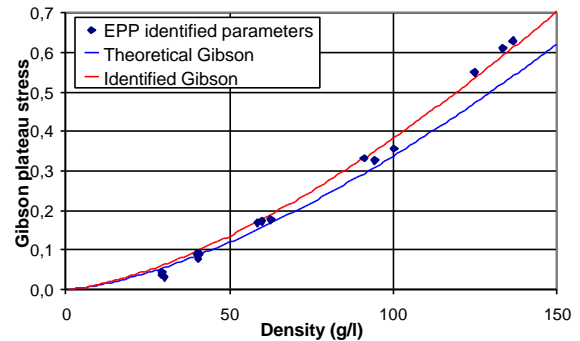
For open cell foams with plastic collapse behaviour the variation of the plastic collapse stress with density is modelled by Gibson with the following relation:

$$\frac{s_y}{s_{yS}} \approx 0.3 \left( \frac{r}{r_s} \right)^{\frac{3}{2}} \quad (10)$$

$s_y$  and  $s_{yS}$  are the plastic collapse stress of the foam and the yield stress of the solid material respectively. As made for the elastic modulus the equation can be written in the following form:

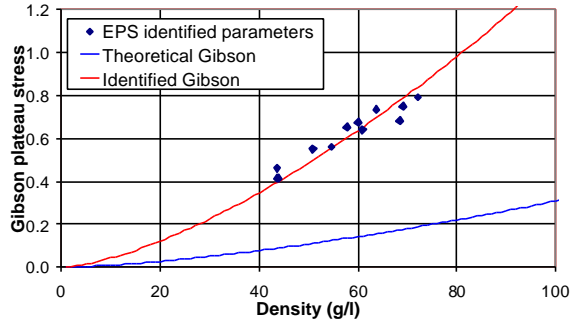
$$s_y = C_y r^{\frac{3}{2}} \quad (11)$$

The parameter can be identified through the experimentally identified plastic collapse stress for each tested foam. The identified plastic collapse stresses and the identified previous relations are shown in Figures 15-18 for each type of foam.

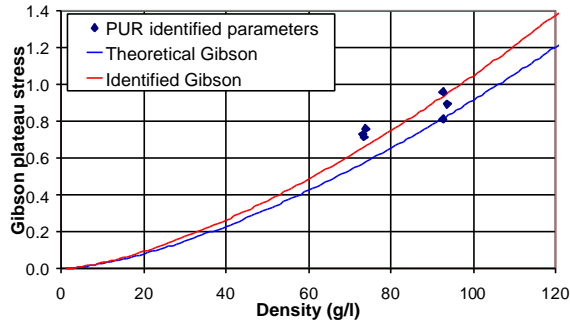


**Figure 15. Gibson model for the EPP foams: identified plateau stress parameters, theoretical parameter-density relation and identified relation.**

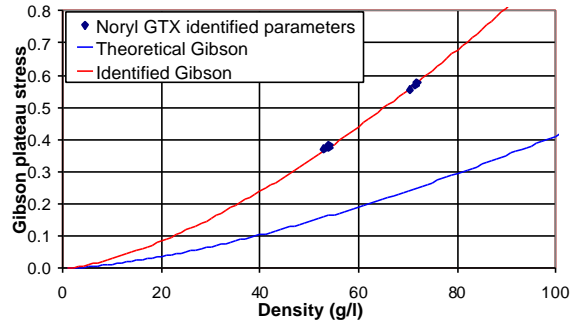




**Figure 16. Gibson model for the EPS foams: identified plateau stress parameters, theoretical parameter-density relation and identified relation.**



**Figure 17. Gibson model for the PUR foams: identified plateau stress parameters, theoretical parameter-density relation and identified relation.**



**Figure 18. Gibson model for the Noryl GTX foams: identified plateau stress parameters, theoretical parameter-density relation and identified relation**

### Densification Strain

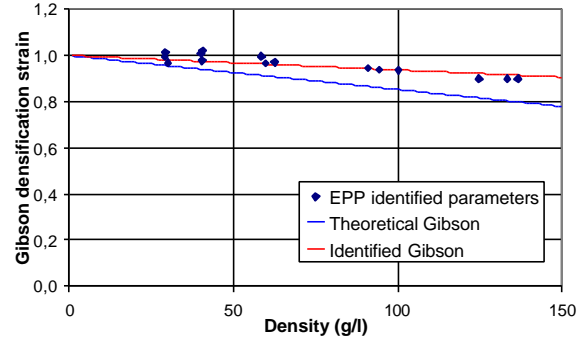
For the densification strain parameter Gibson proposes the equation

$$e_D = 1 - 1.4 \left( \frac{r}{r_s} \right) \quad (12)$$

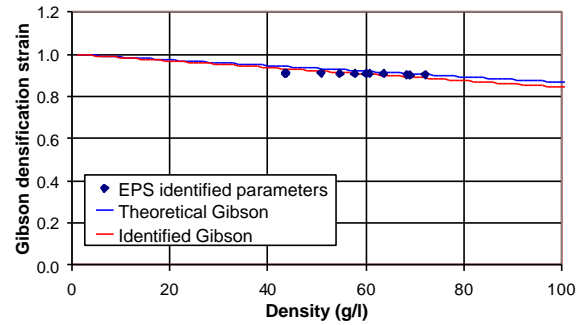
This is not derived directly from micromechanical mechanisms; it is defined with a semi-empirical approach, so that the value 1.4 can be substituted by a constant to be identified:

$$e_D = 1 - C_D r \quad (13)$$

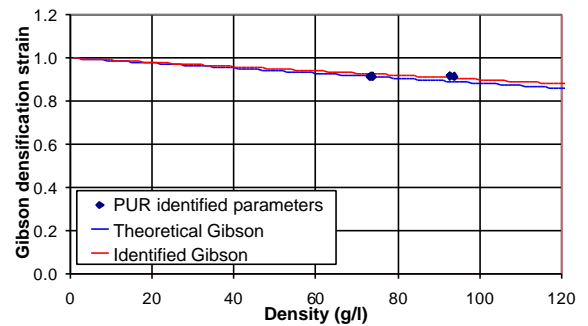
The identified densification strains and the identified previous relations are shown in Figures 19-22 for each type of foam.



**Figure 19. Gibson model for the EPP foams: identified densification strain parameters, theoretical parameter-density relation and identified relation.**

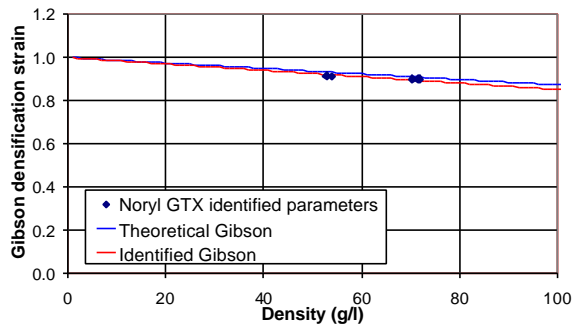


**Figure 20. Gibson model for the EPS foams: identified densification strain parameters, theoretical parameter-density relation and identified relation.**



**Figure 21. Gibson model for the PUR foams: identified densification strain parameters,**

## theoretical parameter-density relation and identified relation.



**Figure 22. Gibson model for the Noryl GTX foams: identified densification strain parameters, theoretical parameter-density relation and identified relation.**

### Density Independent Parameters

The parameter  $D$  and  $m$  are considered density independent and Gibson suggests the values  $D=2.3$  and  $m=1\pm0.4$  for the plastic collapse foams.

In this work a unique value of each parameter has been identified directly from the experimental data of the whole set of tested foams of the same type. The identified values for each kind of foam are shown in Table 1.

**Table 1.**  
**Identified density independent parameters.**

foam	$D$	$m$
EPP	1.01	1.29
EPS	1.34	0.73
PUR	2.73	1.07
Noryl GTX	0.84	0.73

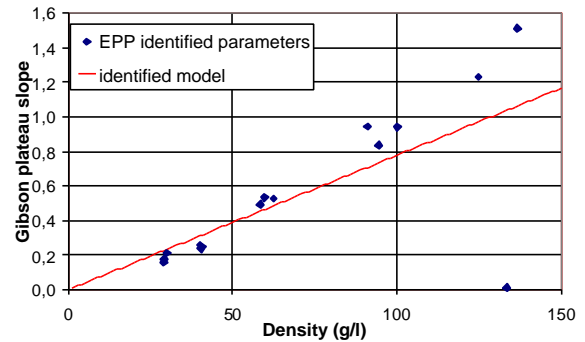
## 6.2 Modified Gibson Model

As for the original Gibson model the parameters-density laws for the modified Gibson model have been identified and compared with the theoretical Gibson laws. In this case a new parameter is analysed: the slope of the plateau stress.

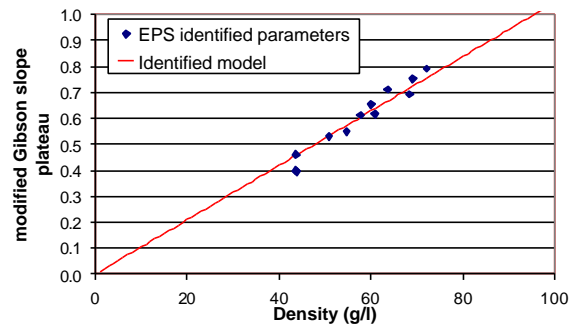
The identified values of this parameter for the tested foams are highly dispersed at higher density values and for certain type of foam. In some cases they are not significant. This could be caused by the fact that the plateau region is limited for higher density foams till to be completely excluded from the model. The identified value for an EPP specimen at 145 g/l is nearly zero: the model in this case shows a shift from the linear region directly to the densification region. The same thing happens for all

Noryl GTX foams where the plateau region is completely unused and its slope remains at the initially assigned value: this is confirmed by the identical behaviour of the Gibson model and the modified Gibson model for this type of foam.

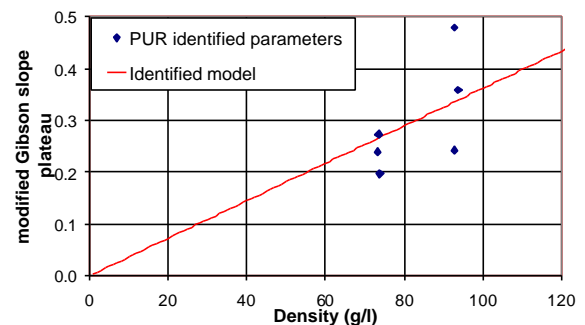
Because of this behaviour a density dependence law is difficult to be defined for this parameter and a simple linear law has been identified. The identified curves for each kind of foam are shown in Figures 23-26.



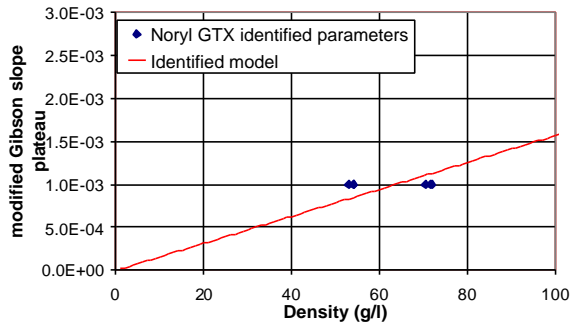
**Figure 23. Linear density dependence law of the slope  $h$  parameter identified for the EPP foams.**



**Figure 24. Linear density dependence law of the slope  $h$  parameter identified for the EPS foams.**



**Figure 25. Linear density dependence law of the slope  $h$  parameter identified for the PUR foams.**



**Figure 26.** Linear density dependence law of the slope  $h$  parameter identified for the Noryl GTX foams.

## 7. MODELS AS A DESIGN TOOL

Test results can be shown in different forms depending on the foam characteristics to be studied and the design purposes. The effects of the foam density have been underlined as the first design parameter to be chosen.

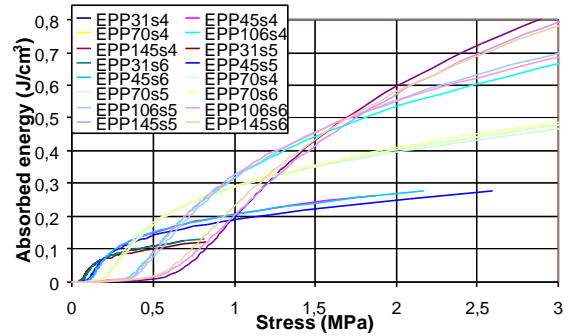
Force-displacement and stress-strain curves, directly derived from the experimental data, show the capability of cellular solids to absorb a high quantity of energy maintaining a nearly constant level of stress in the so called plateau region, but they do not allow directly selecting the foam density suitable to the designed maximum stress and energy amount to be absorbed. The energy versus stress curves are more useful to this aim.

The use of an efficiency coefficient is an even more interesting solution to show the optimal impact loading conditions for defined foam. The efficiency of foam in impact absorption is defined as the ratio of the absorbed energy to the maximum stress value. Each foam should be used on the impact energy and maximum stress value defined by its maximum efficiency conditions [5-6].

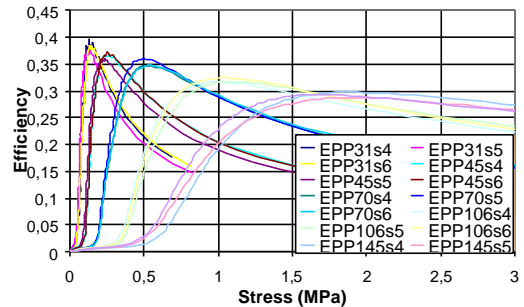
The proper foam density has to be chosen in order to reach the maximum efficiency on the basis of the defined impact energy to be absorbed and the maximum acceptable stress level. It has to be noted that the foam density with the maximum efficiency for the defined application, corresponds to the foam density which reaches the maximum absorption of energy with the defined maximum level of stress or, likewise, to the foam density which need the minimum stress level to absorb the defined impact energy.

Energy-stress curves, for the EPP foams, and efficiency-stress and efficiency-density curves, for all tested materials, are shown in Figures 27-35. The experimental diagrams that express a given quantity as function of the density result in a set of test points

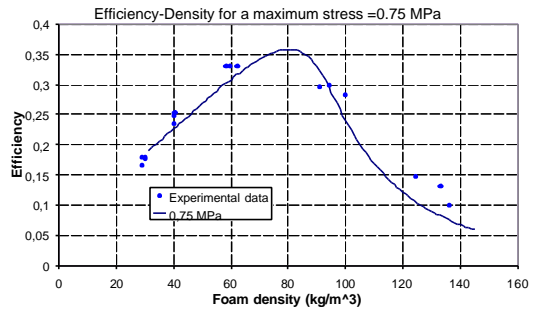
and not in continuous curves. It is not feasible to examine the density at too many levels.



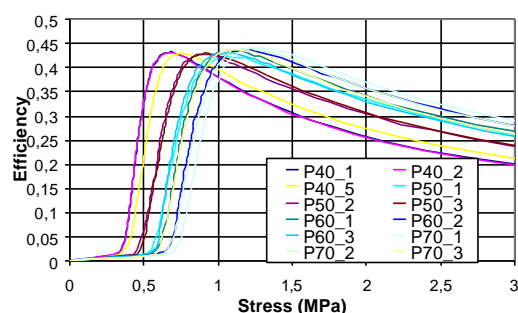
**Figure 27.** Specific energy-stress experimental curves for the EPP foams.



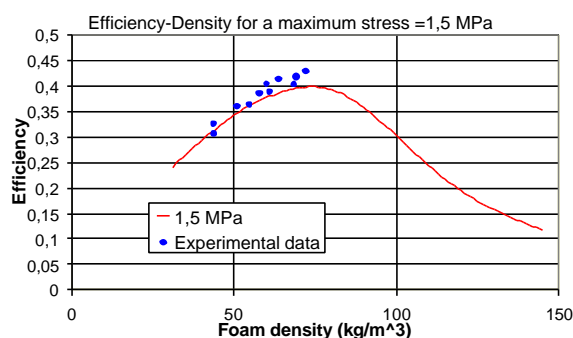
**Figure 28.** Experimental efficiency-stress curves for EPP foams.



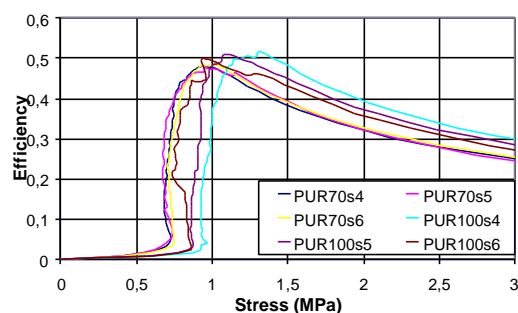
**Figure 29.** Experimental efficiency points at a defined stress of 0.75 MPa and model curve for EPP foams.



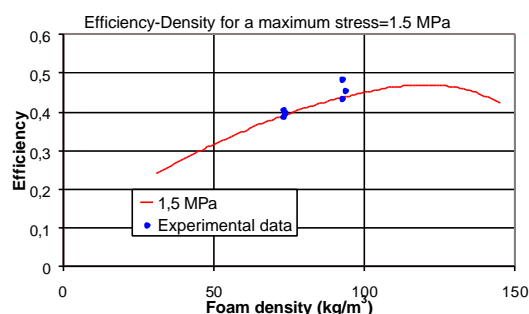
**Figure 30. Experimental efficiency-stress curves for EPS foams.**



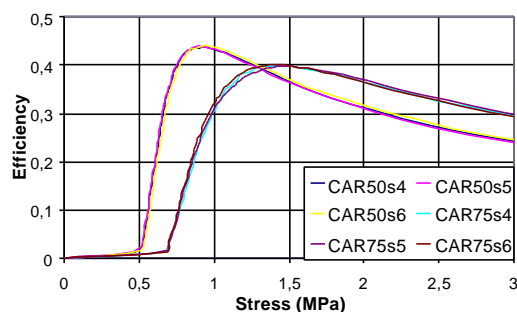
**Figure 31. Experimental efficiency points at a defined stress of 1.5 MPa and model curve for EPS foams.**



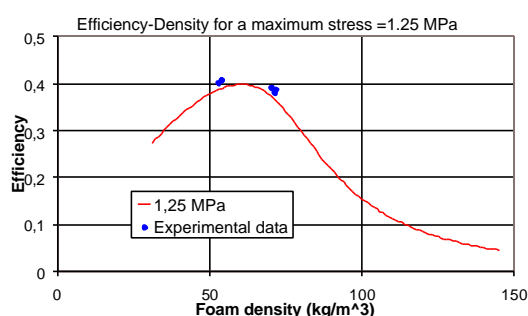
**Figure 32. Experimental efficiency-stress curves for PUR foams.**



**Figure 33. Experimental efficiency points at a defined stress of 1.5 MPa and model curve for PUR foams.**

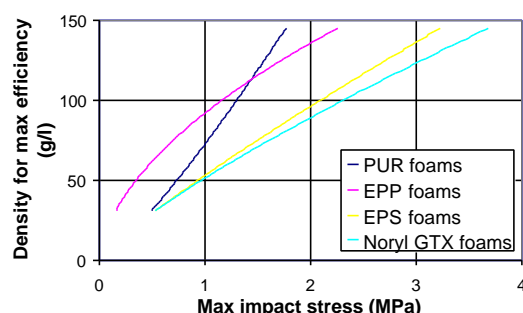


**Figure 34. Experimental efficiency-stress curves for Noryl GTX foams.**

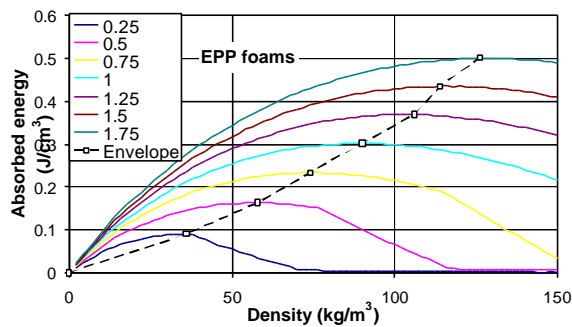


**Figure 35. Experimental efficiency points at a defined stress of 1.25 MPa and model curve for Noryl GTX foams.**

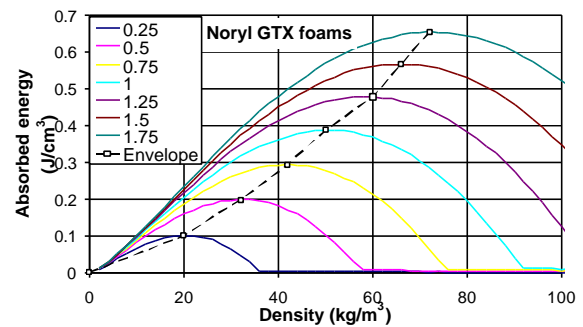
Modelling can be used to simplify the foam selection. The identified models with its parameters-density laws bring a general model of all possible foams obtained from the same constitutive material and same microstructure. This allows predicting the mechanical behaviour of foam of any density with a minimum set of experimental tests.



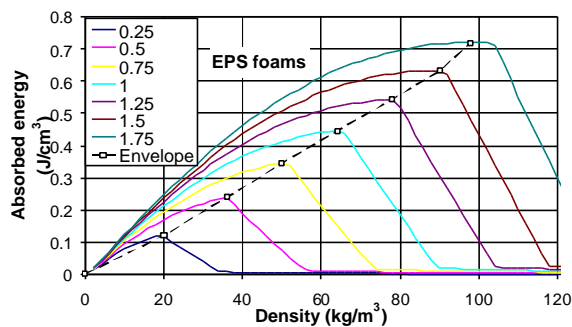
**Figure 36. Modeled optimal density versus maximum stress curves for all tested foams.**



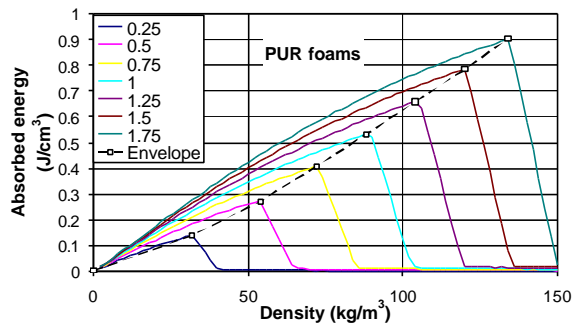
**Figure 37. Modeled (modified Gibson) specific absorbed energy versus density curves for different stress level and envelope of their maximum for EPP foams.**



**Figure 40. Modeled (modified Gibson) specific absorbed energy versus density curves for different stress level and envelope of their maximum for Noryl GTX foams.**



**Figure 38. Modeled (modified Gibson) specific absorbed energy versus density curves for different stress level and envelope of their maximum for EPS foams.**



**Figure 39. Modeled (modified Gibson) specific absorbed energy versus density curves for different stress level and envelope of their maximum for PUR foams.**

Moreover, modelling allows evaluating the energy-stress, energy-density, stress-density, efficiency-stress and efficiency-density curves for any maximum level of stress or absorbed energy. The efficiency-density curves of all tested foams at a defined stress value have been drawn over the experimental data in previous diagrams. These curves clarify the possibility of the models to precisely evaluate the optimal foam density even at the limits of the experimental domain or between two relatively different experimental values of the density.

By means of the identified modified Gibson model and with the help of automated routines, various diagrams have been constructed in order to help foam component design. If the stress limit (acceleration limit) is the main objective of the design, density versus stress curves are to be used. These model-based curves are shown in Figure 36 for the tested materials. These curves bring out the foam density value that allows for the maximum efficiency while maintaining a defined maximum stress. These diagrams lead to the choice of the optimal density, but do not show the energy involved: the dimensions of the foam have to be chosen with an energy diagram.

A diagram which incorporates all the design parameters must contain more curves. For example, several specific absorbed energy versus density curves for a wide range of maximum stress values combine all information about a specific kind of foam. This kind of diagrams have been constructed on the basis of the modified Gibson model and are shown in Figures 37-40 for each analysed type of foam.

The modified version of the Gibson model compared to the original Gibson model has demonstrated to be more suitable to be used in numerical procedures for the construction of proposed design diagrams. It has the advantage to be strictly increasing and consequently to have a unique

strain value for a defined stress. On the contrary the original Gibson model has an undefined value of strain corresponding to the plateau stress and it can cause problem in numerical procedures definition.

## 8. CONCLUSIONS

All analysed and identified cellular solid models fit well the experimental curves of the four kinds of foam tested. The Gibson Model and the modified version of this model show a better fitting capability compared to the Rusch model and the modified Rusch model. The identification, by means of optimisation procedures, has been slower because of the three different formulations for each stress-strain region.

The micro-mechanical density dependence laws of the Gibson model have been identified by means of the experimentally obtained parameters for different densities of the same type of foam. These laws have been compared to the simplified relations suggested by Gibson and in some cases have shown to be significantly different.

The density dependence laws have been identified also for the modified Gibson model. For the newly introduced parameter, the slope of the plateau region, a linear density dependence law has been considered.

The density dependence laws combined with the foam models permit complete modelling of a certain type of foam on a wide range of density, by testing very few values of density. This is useful for an effective choice of the proper foam density for a specific application. Energy diagrams, efficiency curves (shown in [5]) and any other kind of diagrams that describe the effect of density could be obtained by means of the modified Gibson model and few experimental tests. This kind of modelling has shown to be an efficient tool in optimal design of impact absorbers for passive safety of vehicles.

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